Class IX Session 2023-24
Subject - Mathematics
Sample Question Paper- 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. A point is at a distance of 3 units from the x-axis and 7 units from the y-axis. Which of the following may be the co-ordinates of the point? **[1]**

a) (7, 3) b) (3, 7)

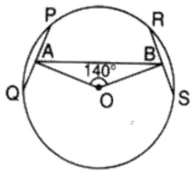
c) (4, 5) d) (0, 0)

2. Each side of an equilateral triangle measures 8 cm. The area of the triangle is **[1]**

a) $32\sqrt{3}\text{cm}^2$ b) $48\ \text{cm}^2$

c) $16\sqrt{3}\text{cm}^2$ d) $8\sqrt{3}\text{cm}^2$

3. In the given figure PQ and RS are two equal chords of a circle with centre O. OA and OB are perpendiculars on chords PQ and RS, respectively. If $\angle AOB = 140^o$, then $\angle PAB$ is equal to **[1]**



a) 60^o b) 70^o

c) 40^o d) 50^o

4. In fig ABCD is a parallelogram. If $\angle DAB = 60^\circ$ and $\angle DBC = 80^\circ$ then $\angle CDB$ is **[1]**

a) 60°

b) 40°

c) 70°

d) 80°

5. The value of **x** in $3 + 2^x = (64)^{\frac{1}{2}} + (27)^{\frac{1}{3}}$ is **[1]**

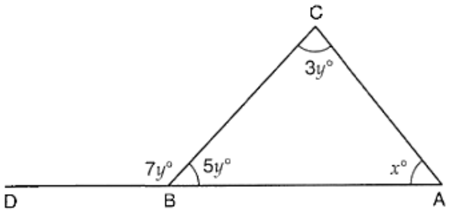
a) 14

b) 8

c) 5

d) 3

6. In figure, what is the value of x? **[1]**



a) 60

b) 35

c) 45

d) 50

7. Write the linear equation such that each point on its graph has an ordinate 5 times its abscissa. **[1]**

a) y = 5x

b) none of these

c) 5x + y = 2

d) x = 5y

8. $\sqrt{3}$ is a polynomial of degree. **[1]**

a) 0

b) 2

c) $\frac{1}{2}$

d) 1

9. If g = $t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$, what is the value of g when t = 64? **[1]**

a) $\frac{31}{2}$

b) $\frac{257}{16}$

c) $\frac{33}{2}$

d) 16

10. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if **[1]**

a) ABCD is a Rhombus

b) Diagonals of ABCD are equal and perpendicular

c) Diagonals of ABCD are perpendicular

d) Diagonals of ABCD are equal

11. $(125)^{-1/3} = ?$ **[1]**

a) $-\frac{1}{5}$

b) -5

c) $\frac{1}{5}$

d) 5

12. Any solution of the linear equation 2x + 0y + 9 = 0 in two variables is of the form **[1]**

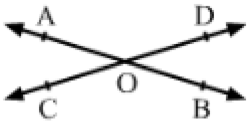
a) $\left(-\frac{9}{2}, m\right)$

b) (-9 , 0)

c) $\left(0, -\frac{9}{2}\right)$

d) $\left(n, -\frac{9}{2}\right)$

13. In the given figure, straight lines AB and CD intersect at O. If $\angle AOC + \angle BOD = 130^\circ$ then $\angle AOD = ?$ **[1]**



a) 110°

b) 65°

c) 115°

d) 125°

14. If $\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$, then x = **[1]**

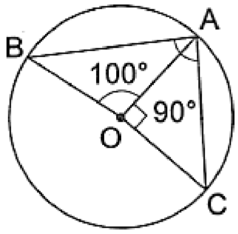
a) 4

b) 2

c) 5

d) 3

15. In the given figure, O is the centre of a circle. If ∠AOB = 100° and ∠AOC = 90°, then ∠BAC = ? **[1]**



a) 75°

b) 95°

c) 85°

d) 80°

16. If (x, y) = (y, x), then **[1]**

a) x – y = 0

b) x + y = 0

c) x ÷ y = 0

d) xy = 0

17. If a linear equation has solutions (1, 2), (-1, -16) and (0, -7), then it is of the form **[1]**

a) y = 9x - 7

b) 9x - y + 7 = 0

c) x - 9y = 7

d) x = 9y - 7

18. The value of $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$ is **[1]**

a) $x^4 + \frac{1}{x^4}$

b) $x^3 - \frac{1}{x^3} + 2$

c) $x^2 + \frac{1}{x^2} - 2$

d) $x^4 - \frac{1}{x^4}$

19. **Assertion (A):** In ΔABC, E and F are the midpoints of AC and AB respectively. The altitude AP at BC intersects FE at Q. Then, AQ = QP. **[1]**

Reason (R): Q is the midpoint of AP.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Every integer is a rational number. **[1]**

Reason (R): Every integer m can be expressed in the form $\frac{m}{l}$.

a) Both A and R are true and R is the correct explanation of A.

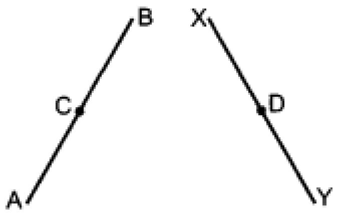
b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

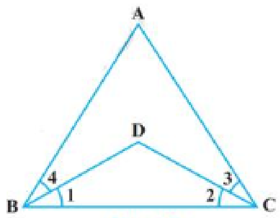
d) A is false but R is true.

Section B

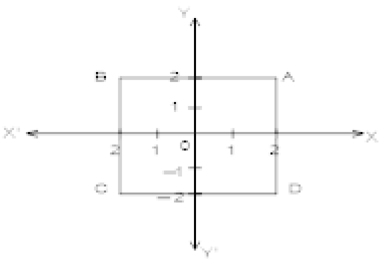
21. In fig. AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using a Euclid's axiom, show that AB = XY. **[2]**



22. In the given figure, we have $\angle ABC = \angle ACB, \angle 4 = \angle 3$. Show that $\angle 1 = \angle 2$. **[2]**



23. Find Co-ordinates of vertices of rectangle ABCD. **[2]**



24. Assuming that x is a positive real number and a, b, c are rational numbers, show that: **[2]**

$\left(\frac{x^a}{x^b}\right)^{a+b}\left(\frac{x^b}{x^c}\right)^{b+c}\left(\frac{x^c}{x^a}\right)^{c+a}$ = 1

OR

Express the decimal $18.\overline{48}$ in the form $\frac{p}{q}$, where p, q are integers and q $\neq$ 0.

25. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights. **[2]**

OR

The surface areas of two spheres are in the ratio of 4 : 25. Find the ratio of their volumes.

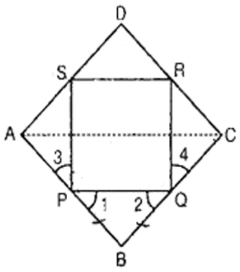
Section C

26. Simplify $\left\{\left[625^{\frac{-1}{2}}\right]^{-\frac{1}{4}}\right\}^2$ **[3]**

27. Draw a histogram of the following distribution: **[3]**

Height (in cm)	Number of students
150 - 153	7
153 - 156	8
156 - 159	14
159 - 162	10
162 - 165	6
165 - 168	5

28. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle. **[3]**



29. Find at least 3 solutions for the following linear equation in two variables: 5x + 3y = 4. **[3]**

30. The monthly profits (in Rs) of 100 shops are distributed as follows: **[3]**

Profits per shop:	0-50	50-100	100-50	150-200	200-250	250-300
No. of shops:	12	18	27	20	17	6

Draw a histogram for the data and show the frequency polygon for it.

OR

Draw a frequency polygon for the following distribution:

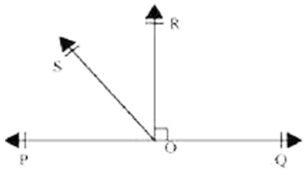
Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	7	10	6	8	12	3	2	2

31. Factorize the polynomial: **[3]**

$8a^3 - b^3 - 12a^2b + 6ab^2$

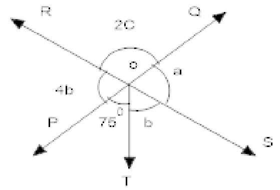
Section D

32. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$. **[5]**



OR

In fig two straight lines PQ and RS intersect each other at O, if ∠POT = 75° Find the values of a, b and c



33. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm^2. **[5]**

34. The perimeter of a right triangle is 24 cm. If its hypotenuse is 10 cm, find the other two sides. Find its area by using the formula area of a right triangle. Verify your result by using Heron's formula. **[5]**

OR

Find the area of the triangle whose sides are 42 cm, 34 cm and 20 cm in length. Hence, find the height corresponding to the longest side.

35. If $(x^3 + ax^2 + bx + 6)$ has (x - 2) as a factor and leaves a remainder 3 when divided by (x - 3), find the values of a and b. **[5]**

Section E

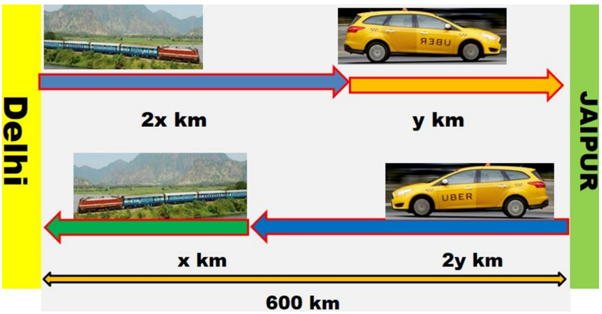
36. **Read the text carefully and answer the questions:** **[4]**

Ajay lives in Delhi, The city of Ajay's father in laws residence is at Jaipur is 600 km from Delhi. Ajay used to travel this 600 km partly by train and partly by car.

He used to buy cheap items from Delhi and sale at Jaipur and also buying cheap items from Jaipur and sale at Delhi.

Once From **Delhi to Jaipur** in forward journey he covered 2x km by train and the rest y km by taxi.

But, while returning he did not get a reservation from Jaipur in the train. So first 2y km he had to travel by taxi and the rest x km by Train. From Delhi to Jaipur he took 8 hrs but in returning it took 10 hrs.



(i) Write the above information in terms of equation.

(ii) Find the value of x and y?

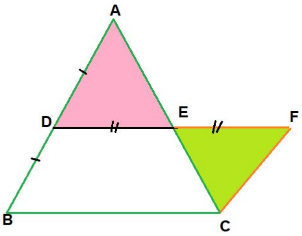
(iii) Find the speed of Taxi?

OR

Find the speed of Train?

37. **Read the text carefully and answer the questions:** **[4]**

Haresh and Deep were trying to prove a theorem. For this they did the following



i. Draw a triangle ABC

ii. D and E are found as the mid points of AB and AC

iii. DE was joined and DE was extended to F so DE = EF

iv. FC was joined.

(i) $\triangle$ADE and $\triangle$EFC are congruent by which criteria?

(ii) Show that CF$\|$ AB.

(iii) Show that CF = BD.

OR

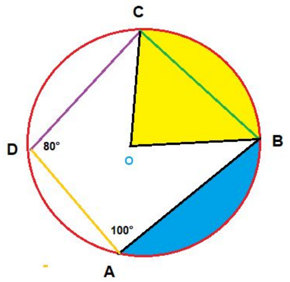
Show that DF = BC and DF$\|$ BC.

38. **Read the text carefully and answer the questions:** **[4]**

There was a circular park in Defence colony at Delhi. For fencing purpose poles A, B, C and D were installed at the circumference of the park.

Ram tied wires From A to B, B to C and C to D, and he managed to measure the $\angle A = 100°$ and $\angle D = 80°$

Point O in the middle of the park is the center of the circle.



(i) Name the quadrilateral ABCD.

(ii) What is the value of $\angle$C?

(iii) What is the value of $\angle B$.

OR

Write any three properties of cyclic quadrilateral?

Solution

Section A

1. **(a)** (7, 3)

Explanation: We know that distance of any point from x-axis is the y-ordinate, so here y-coordinate = 3.

Now, distance of any point from y-axis is the x coordinate of the point.

So, here x co-ordinate is = 7

Thus, point will be (7, 3)

2.

(c) $16\sqrt{3}\text{cm}^2$

Explanation: Area of equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{ Side })^2$

$=\frac{\sqrt{3}}{4}\times(8)^2$

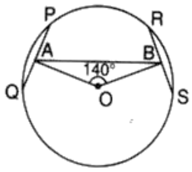
$=\frac{\sqrt{3}}{4}\times 64$

$=16\sqrt{3}\text{cm}^2$

3.

(b) 70^o

Explanation:



In triangle ABO, AO = BO

So, $\angle BAO=\angle ABO=x$

$x+x+140^0=180^0$

$\Rightarrow 2x=40^0$

$x=20^0$

Now $\angle QAO+\angle BAO+\angle PAB=180^0$

Substituting the values we get:-

$90^0+20^0+\angle PAB=180^0$

$\angle PAB=70^0$

4.

(b) 40°

Explanation: 40° Angle C = 60° as opposite angles of a parallelogram are equal and angle CDB = 40° angle sum property of a triangle. [In triangle CDB, angle C + angle CDB + angle DBC = 180°]

5.

(d) 3

Explanation: $3 + 2^x = (64)^{\frac{1}{2}}+(27)^{\frac{1}{3}}$

$\Rightarrow 3 + 2^x = \sqrt{64}+\sqrt[3]{27}$

$\Rightarrow 3 + 2^x = 8 + 3$

$\Rightarrow 2^x = 8 = 2^3$

equating both,

x = 3

6. **(a)** 60

Explanation: In $\triangle$ABC,

$\angle$BCA + $\angle$CAB + $\angle$ABC = 180°

$\Rightarrow 3y^\circ + x^0 + 5y^\circ$ = 180°
$\Rightarrow 8y^\circ + x^\circ$ = 180° ....(i)
Also, $5y^\circ$ = 180° - $7y^\circ$
$\Rightarrow 12y^\circ = 180^\circ$
$\Rightarrow y^\circ = 15^\circ$
From (i), $x^0 = 180^\circ - 8y^\circ$
$\Rightarrow x^0 = 180^\circ - 8 \times 15^\circ$
$\Rightarrow x^0 = 60^\circ$

7. **(a)** y = 5x
Explanation: y = 5x

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at x = 1
y = 5.1 = 5
y = 5
(1,5)

---

at x = 2
y = 5.2 = 10
y = 10
(2,10)

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at x = 3
y = 5.3 = 15
y = 15
(3,15)

8. **(a)** 0
Explanation: $\sqrt{3}$ is a constant term, so it is a polynomial of degree 0.

9. **(c)** $\frac{33}{2}$
Explanation: g = $t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$
= $t^{\frac{2}{3}} + 4 \times \frac{1}{t^{\frac{1}{2}}}$
= $(64)^{\frac{2}{3}} + 4 \times \frac{1}{64^{\frac{1}{2}}}$
= $\left(4^3\right)^{\frac{2}{3}} + 4 \times \frac{1}{\left(8^2\right)^{\frac{1}{2}}}$
= $4^{\frac{2}{3} \times 3} + 4 \times \frac{1}{8^{2 \times \frac{1}{2}}}$
= $4^2 + \frac{4}{8}$
= $16 + \frac{1}{2}$
= $\frac{33}{2}$

10. **(b)** Diagonals of ABCD are equal and perpendicular
Explanation: A quadrilateral formed by joining the mid points of a square is a square. So, ABCD is a square. In Square, diagonals are equal and perpendicular.

11. **(c)** $\frac{1}{5}$
Explanation: $(125)^{-1/3}$
=$(5^3)^{-1/3}$
= 5^{-1}
$\frac{1}{5}$

12. **(a)** $\left(-\frac{9}{2}, m\right)$

Explanation: 2x + 9 = 0

$\Rightarrow x = \frac{-9}{2}$ and y = m, where m is any real number

Hence, $\left(-\frac{9}{2}, m\right)$ is the solution of the given equation.

13.

(c) 115°

Explanation: We have:

$\angle AOC = \angle BOD$ [Vertically-Opposite Angles]

$\therefore \angle AOC + \angle BOD = 130^\circ$

$\Rightarrow \angle AOC + \angle AOC = 130^\circ$ $[\because \angle AOC = \angle BOD]$

$\Rightarrow 2\angle AOC = 130^\circ$

$\Rightarrow \angle AOC = 65^\circ$

Now,

$\angle AOC + \angle AOD = 180^\circ$ [∵ COD is a straight line]

$\Rightarrow 65^\circ + \angle AOD = 180^\circ$

$\Rightarrow \angle AOC = 115^\circ$

14.

(c) 5

Explanation: $\frac{3^{2x-8}}{225} = \frac{5^3}{5^x}$

$\Rightarrow 3^{2x-8} \times 5^x = 5^3 \times 225$

$\Rightarrow \frac{3^{2x}}{3^8} \times 5^x = 5^3 \times 5 \times 5 \times 3 \times 3$

$\Rightarrow 3^{2x} \times 5^x = 3^8 \times 3^2 \times 5^5$

$\Rightarrow (3^2)^x \times 5^x = 3^{10} \times 5^5$

$\Rightarrow 9^x \times 5^x = 9^5 \times 5^5$

$\Rightarrow (45)^x = (45)^5$

Comparing, we get

x = 5

15.

(c) 85°

Explanation: We have:

∠BOC + ∠BOA + ∠AOC = 360°

⇒ ∠BOC + 100° + 90° = 360°

⇒ ∠BOC = (360° - 190°) = 170°

$\therefore \angle \text{BAC} = \left(\frac{1}{2} \times \angle \text{BOC}\right) = \left(\frac{1}{2} \times 170^\circ\right) = 85^\circ$

$\Rightarrow \angle \text{BAC} = 85^o$

16. **(a)** x – y = 0

Explanation: If (x,y) = (y,x),

It means abscissa =ordinate or, x=y

So,

X—Y = 0 {since x=y,}

17. **(a)** y = 9x - 7

Explanation: Since all the given co- ordinate (1, 2), (-1, -16) and (0, -7) satisfy the given line y = 9x - 7

For point (1, 2)

y = 9x - 7

2 = 9(1) - 7

2 = 9 - 7

2 = 2

Hence (2, 1) is a solution.

For point (-1, -16)

y = 9x - 7

-16 = 9(-1) - 7

-16 = -9 - 7

-16 = -16

Hence (-1, -16) is a solution.

For point (0,-7)

y = 9x - 7

-7 = 9(0) -7

-7 = -7

Hence (0, -7) is a solution.

18.

(d) $x^4 - \frac{1}{x^4}$

Explanation: $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)$

= $\left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)$ [Using identity (a + b)(a - b) = $a^2 - b^2$]

= $x^4 - \frac{1}{x^4}$ [Using identity (a + b) (a - b) = $a^2 - b^2$]

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

In ΔABC, E and F are midpoint of the sides AC and AB respectively.

FE || BC [By mid-point theorem]

Now, in ΔABP, F is mid-point of AB and FQ || BP. Q is mid-point of AP

AQ = QP

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. In the above figure, we have

AB = AC + BC = AC + AC = 2AC (Since, C is the mid-point of AB) ..(1)

XY = XD + DY = XD + XD = 2XD (Since, D is the mid-point of XY) ..(2)

Also, AC = XD (Given) ..(3)

From (1),(2)and(3), we get

AB = XY, According to Euclid, things which are double of the same things are equal to one another.

22. We have

$\Rightarrow \angle ABC = \angle ACB$ ...(1) [(Given)]

And $\angle 4 = \angle 3$ ...(2) [(Given)]

Now, subtracting (2) from (1), we get

Now, by Euclid's axiom 3, if equals are subtracted from equals, the remainders are equal.

$\angle ABC - \angle 4 = \angle ACB - \angle 3$

Hence, $\angle 1 = \angle 2$.

23. The co- ordinates of vertices of rectangle A (2, 2), B (-2, 2), C (-2, -2) and D (2, -2). it is a square.

24. $\left(\frac{x^a}{x^b}\right)^{a+b}\left(\frac{x^b}{x^c}\right)^{b+c}\left(\frac{x^c}{x^a}\right)^{c+a}$

= $(x^{a-b})^{a+b}(x^{b-c})^{b+c}(x^{c-a})^{c+a}$ [using $\frac{a^m}{a^n} = a^{m-n}$]

= $x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

= $x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

= $x^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= x^0 = 1$

Hence proved.

OR

Let x $= 18.\overline{48}$

i.e. x = 18.4848 ….(i)

Multiply eq. (i) by 100 we get

⇒ 100x = 1848.4848 ….(ii)

On subtracting (i) from (ii), we get

99x = 1830

$\Rightarrow x = \frac{1830}{99}$

$\Rightarrow x = \frac{610}{33}$

25. Let the radius of base of hemisphere and cone, each be r cm. Let the height of the cone be h cm.

Volume of the cone = $\frac{1}{3}\pi r^2$ hcm^3

Volume of the hemisphere = $\frac{2}{3}\pi r^3$ cm^3

According to the question, $\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

⇒ h = 2r

⇒ Height of the cone = 2r cm.

Height of the hemisphere = r cm

∴ Ratio of their heights = 2r : r = 2 : 1

OR

Surface areas of two spheres $= \frac{4}{25}$

$\Rightarrow \frac{4R^2}{4r^2} = \frac{4}{25}$

$\Rightarrow \frac{R^2}{r^2} = \frac{4}{25}$

$\Rightarrow \frac{R}{r} = \frac{2}{5}$

Ratio of their volumes $= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$

$= \left(\frac{R}{r}\right)^3$

$= \left(\frac{2}{5}\right)^3$

$= \frac{8}{125}$

Hence, the ratio of their volumes is 8:125

Section C

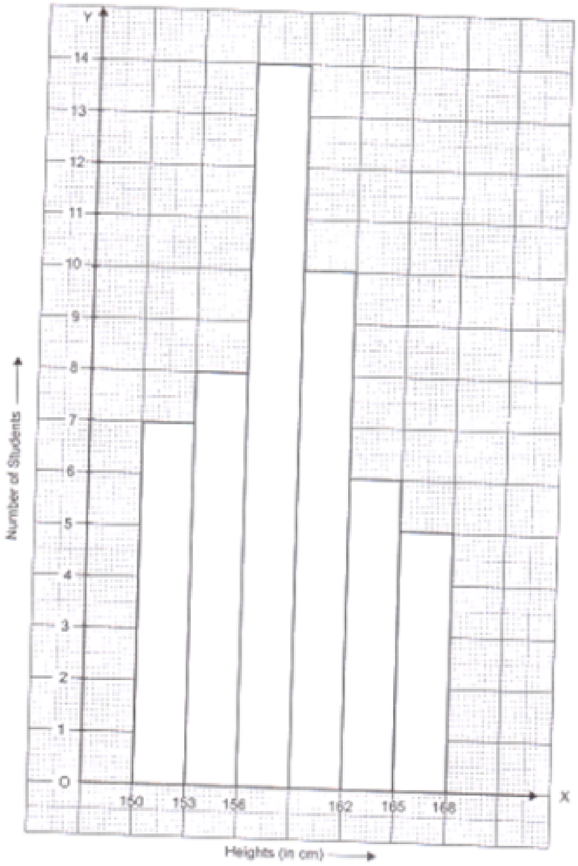
26. $\left\{\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right\}^2$

$= \left\{\left(\frac{1}{625^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2 = \left\{\left(\frac{1}{(25^2)^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2 \left(a^{-m} = \frac{1}{a^m}\right)$

$= \left\{\left(\frac{1}{25}\right)^{-\frac{1}{4}\times 2}\right\}$

$= \left(\frac{1}{25^{-\frac{1}{2}}}\right) = \frac{1}{(5^2)^{-\frac{1}{2}}} = \frac{1}{5^{-1}} = 5$

27. Histogram which represent the given frequency distribution is shown below:



28. Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In △ABC, P is the mid-point of AB and Q is the mid-point of BC.

∴ PQ ∥ AC and PQ = $\frac{1}{2}$ AC ...(i)

In △ADC, R is the mid-point of CD and S is the mid-point of AD.

∴ SR ∥ AC and SR = $\frac{1}{2}$ AC ...(ii)

From eq. (i) and (ii),PQ ∥ SR and PQ = SR

∴ PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

∴ AB = BC

⇒ BP = $\frac{1}{2}$ BC ⇒ BP = BQ

∴ ∠1 = ∠2 [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly AS = CR and PS = QR [Opposite sides of a parallelogram]

∴ △ APS ≅ △CQR [By SSS congreuancy]

⇒ ∠3 = ∠4 [By C.P.C.T.]

Now we have ∠1 + ∠SPQ + ∠3 = 180° And ∠2 + ∠PQR + ∠4 = 180° [Linear pairs]

∴ ∠1 + ∠SPQ + ∠3 = ∠2 + ∠PQR + ∠4

Since ∠1 = ∠2 and ∠3 = 4[Proved above]

∴ ∠SPQ = ∠PQR ...(iii)

Now PQRS is a parallelogram [Proved above]

∴ ∠SPQ + ∠PQR = 180° ...(iv) [Interior angles]

Using eq. (iii) and (iv),

∠SPQ + ∠SPQ = 2 ∠SPQ = 180°

⇒ ∠SPQ = 90°

Hence PQRS is a rectangle.

29. 5x + 3y = 4

⇒ 3y = 4 – 5x

$\Rightarrow$ y = $\frac{4-5x}{3}$

put x = 0, then y = $\frac{4-5(0)}{3} = \frac{-4}{3}$

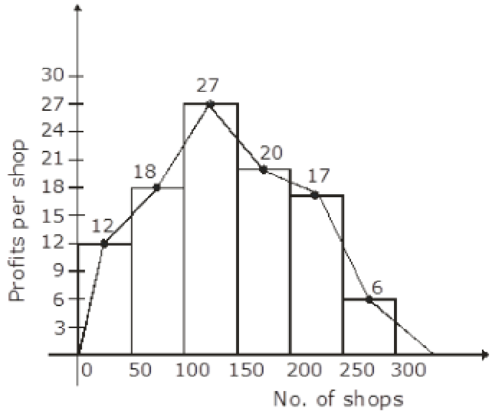
Put x = 1, then y = $\frac{4-5(1)}{3} = -\frac{1}{3}$

Put x = 2, then y = $\frac{4-5(2)}{3}$ = - 2

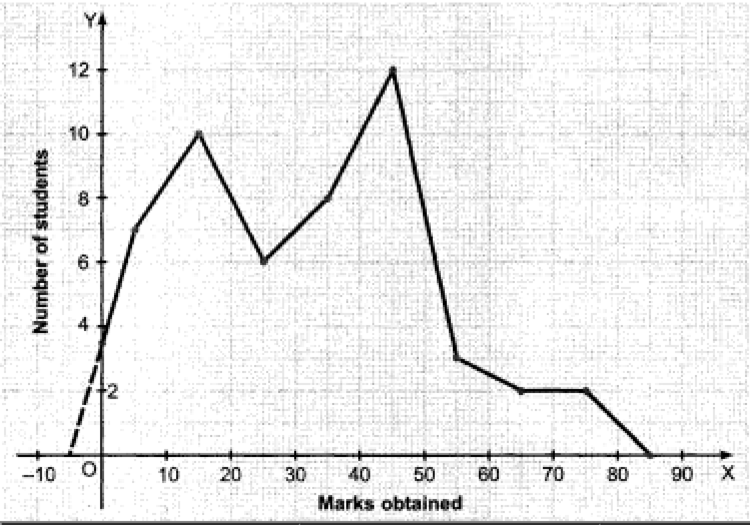
Put x = 3, then y = $\frac{4-5(3)}{3} = \frac{-11}{3}$

$\therefore \left(0, \frac{4}{3}\right), \left(1, \frac{-1}{3}\right)$, (2, -2), and $\left(3, \frac{-11}{3}\right)$ are the solutions of the equation 5x + 3y = 4.

30. Monthly profits (in Rs) of 100 shops



OR



x_i	f_i	(x_i, f_i)
5	7	$(5, 7)$
15	10	$(15, 10)$
25	6	$(25, 6)$
35	8	$(35, 8)$
45	12	$(45, 12)$
55	3	$(55, 3)$
65	2	$(65, 2)$
75	2	$(75, 2)$

31. $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as $= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$.

Using identity $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$ with respect to the expression

$(2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$, we get $(2a - b)^3$

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2, weget (2a - b)^3$

Section D

32. To Prove: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Given: OR is perpendicular to PQ, or ∠QOR = 90°

From the given figure, we can conclude that ∠POR and ∠QOR form a linear pair.

We know that sum of the angles of a linear pair is 180°.

∴ ∠POR + ∠QOR = 180°

or ∠POR = 90°

From the figure, we can conclude that

∠POR = ∠POS + ∠ROS

⇒ ∠POS + ∠ROS = 90°

⇒ ∠ROS = 90° - ∠POS...(i)

Again,

∠QOS + ∠POS = 180°

⇒ $\frac{1}{2}(\angle QOS + \angle POS) = 90^\circ$.(ii)

Substitute (ii) in (i), to get

$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$

$= \frac{1}{2}(\angle QOS - \angle POS)$.

Therefore, the desired result is proved.

OR

PQ intersect RS at O

$\therefore \angle QOS = \angle POR$ [vert'ically opposite angles]

a = 4b ...(1)

Also,

a + b + 75° = 180° [∵POQ is a straight lines]

∴ a + b = 180° - 75°

= 105°

Using, (1)

4b + b = 105°

5b = 105°

Or

b = = $\frac{105^\circ}{5}$ = 21°

Now a=4b

a = 4 × 21°

a = 84°

Again,∠QOR and ∠QOS

∴ a + 2c = 180°

Using, (2) 84° + 2c = 180°

2c = 180° - 84°

2c = 96°

c = $\frac{96^0}{2}$ = 48°c

Hence,

a = 84°, b = 21° and c = 48°

33. Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, 17.6 = $2\pi r$

$2 \times \frac{22}{7} r$ = 17.6 m

So, the radius of the dome = 17.6 $\times \frac{7}{2\times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

= 2 × $\frac{22}{7}$ × 2.8 × 2.8 m^2

= 49.28 m^2

Now, the cost of painting 100 cm^2 is Rs. 5.

So, the cost of painting 1 m^2 = Rs. 500

Therefore, the cost of painting the whole dome

= Rs. 500 $\times$ 49.28

= Rs. 24640

34. Let x and y be the two lines of the right $\angle$

$\therefore$ AB = x cm, BC = y cm and AC = 10 cm

$\therefore$ By the given condition,

Perimeter = 24 cm

x + y +10 = 24 cm

Or x + y = 14 ... (I)

By Pythagoras theorem,

$x^2 + y^2 = (10)^2$ = 100 ... (II)

From (1), $(x+y)^2$ = $(14)^2$

Or $x^2 + y^2$ + 2xy = 196

$\therefore$100 + 2xy =196 [From (II)]

xy =$\frac{96}{2}$= 48 sq cm .... (III)

Area of Δ ABC = $\frac{1}{2}xy$ sq cm

=$\frac{1}{2}$ $\times$ 48 sq cm

=24 sq cm.... (IV)

Again, we know that

$(x\ -\ y)^2 = (x+y)^2 - 4xy$

$=\ (14)^2 - 4 \times 48$ [From (I) & (III)]

Or x - y = $\pm$ 2

(i) When, x-y = 2 and x+y = 14, then 2x = 16

or x = 8, y = 6

(ii) When, X – y = -2 and x + y = 14, then 2x = 12

Or x = 6, y = 8

Verification by using Heron’s formula:

Sides are 6 cm, 8 cm and 10 cm

S =$\frac{24}{2}$ = 12 cm

Area of Δ ABC = $\sqrt{12\ (12-6)\ (12-8)\ (12-10)}$ sq cm

=$\sqrt{12 \times 6 \times 4 \times 2}$ sq cm

= 24 sq cm

Which is same as found in (IV)

Thus, the result is verified.

OR

Let:

a = 42 cm, b = 34 cm and c = 20 cm

$\therefore s = \frac{a+b+c}{2} = \frac{42+34+20}{2} = 48\text{cm}$

By Heron's formula, we have:

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-42)(48-34)(48-20)}$

$= \sqrt{48 \times 6 \times 14 \times 28}$

$= \sqrt{4 \times 2 \times 6 \times 6 \times 7 \times 2 \times 7 \times 4}$

$= 4 \times 2 \times 6 \times 7$

Area of triangle = 336 cm^2

We know that the longest side is 42 cm.

Thus, we can find out the height of the triangle corresponding to 42 cm.

We have:

Area of triangle = 336 cm^2

$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 336

$\Rightarrow \frac{1}{2}$ (42)(height) = 336

$\Rightarrow$ Height $= \frac{336 \times 2}{42}$ = 16 cm

35. Let: $f(x) = x^3 + ax^2 + bx + 6$

$f(x)$ is divisible by $x - 2$

Then $f(2) = 0$

$2^3 + a \times 2^2 + b \times 2 + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a + 2b = -14$

$2a + b = -7$ ...(i)

If $f(x)$ is divided by $x - 3$ remainder is 3

$\therefore f(3) = 3$

$3^3 + a \times 3^2 + b \times 3 + 6 = 3$

$9a + 3b = -30$

$3a + b = -10$ ...(ii)

Subtracting (i) from (ii)

$-a = 3$ and $a = -3$

Put $a = -3$ in eq (i)

$2 \times -3 + b = -7$

$-6 + b = -7$

$b = -7 + 6$

$b = -1$

Section E

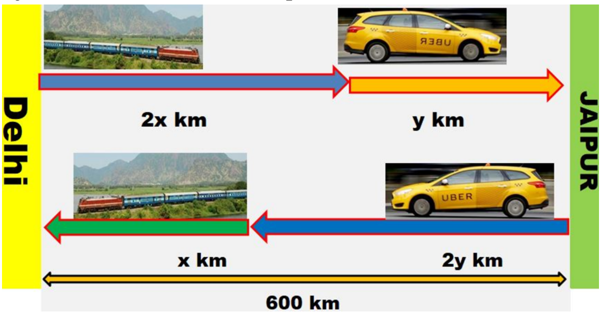
36. **Read the text carefully and answer the questions:**

Ajay lives in Delhi, The city of Ajay's father in laws residence is at Jaipur is 600 km from Delhi. Ajay used to travel this 600 km partly by train and partly by car.

He used to buy cheap items from Delhi and sale at Jaipur and also buying cheap items from Jaipur and sale at Delhi.

Once From **Delhi to Jaipur** in forward journey he covered 2x km by train and the rest y km by taxi.

But, while returning he did not get a reservation from Jaipur in the train. So first 2y km he had to travel by taxi and the rest x km by Train. From Delhi to Jaipur he took 8 hrs but in returning it took 10 hrs.



(i) Delhi to Jaipur: $2x + y = 600$

Jaipur to Delhi: $2y + x = 600$

Let S_1 and S_2 be the speeds of Train and Taxi respectively, then

Dehli to Jaipur: $\frac{2x}{S_1} + \frac{y}{S_2} = 8$ ...(i)

Jaipur to Delhi: $\frac{x}{S_1} + \frac{2y}{S_2} = 10$ ...(ii)

(ii) $2x + y = 600$ ...(1)

$x + 2y = 600$ ...(2)

Solving (1) and (2) $\times$ 2

$2x + y - 2x - 4y = 600 - 1200$

$\Rightarrow -3y = -600$

$\Rightarrow y = 200$

Put $y = 200$ in (1)

$2x + 200 = 600$

$\Rightarrow x = \frac{400}{2} = 200$

(iii) We know that speed $= \frac{Distance}{Time} \Rightarrow$ Time $= \frac{Distance}{Speed}$

Let S_1 and S_2 are speeds of train and taxi respectively.

Delhi to Jaipur: $\frac{2x}{S_1} + \frac{y}{S_2}$ = 8 ...(i)

Jaipur to Delhi: $\frac{x}{S_1} + \frac{2y}{S_2}$ = 10 ...(ii)

Solving (i) and (ii) $\times$ 2

$\Rightarrow \frac{2x}{S_1} + \frac{y}{S_2} - \frac{2x}{S_1} - \frac{4y}{S_2}$ = 8 - 20 = -12

$\Rightarrow \frac{-3y}{S_2}$ = -12

We know that y = 200 km

$\Rightarrow$ S_2 = $\frac{3 \times 200}{12}$ = 50 km/hr

Hence speed of Taxi = 50 km/hr

OR

We know that x = 200 km

Put S_2 = 50 km/hr ...(i)

$\frac{400}{S_1} + \frac{200}{50}$ = 8

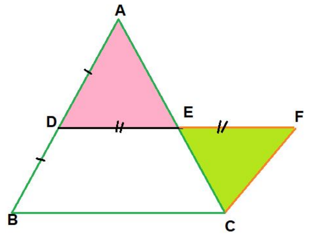
$\Rightarrow \frac{400}{S_1}$ = 8 - 4 = 4

$\Rightarrow$ S_1 = $\frac{400}{4}$ = 100 km/hr

Hence speed of Train = 100 km/hr

37. **Read the text carefully and answer the questions:**

Haresh and Deep were trying to prove a theorem. For this they did the following



i. Draw a triangle ABC
ii. D and E are found as the mid points of AB and AC
iii. DE was joined and DE was extended to F so DE = EF
iv. FC was joined.

(i) $\triangle$ADE and $\triangle$CFE

DE = EF (By construction)

$\angle$AED = $\angle$CEF (Vertically opposite angles)

AE = EC(By construction)

By SAS criteria $\triangle$ADE $\cong$ $\triangle$CFE

(ii) $\triangle$ADE $\cong$ $\triangle$CFE

Corresponding part of congruent triangle are equal

$\angle$EFC = $\angle$EDA

alternate interior angles are equal

$\Rightarrow$ AD $\parallel$ FC

$\Rightarrow$ CF $\parallel$ AB

(iii) $\triangle$ADE $\cong$ $\triangle$CFE

Corresponding part of congruent triangle are equal.

CF = AD

We know that D is mid point AB

$\Rightarrow$ AD = BD

$\Rightarrow$ CF = BD

OR

DE = $\frac{BC}{2}$ {line drawn from mid points of 2 sides of $\triangle$ is parallel and half of third side}

DE $\parallel$ BC and DF $\parallel$ BC

DF = DE + EF

$\Rightarrow$ DF = 2DE(BE = EF)

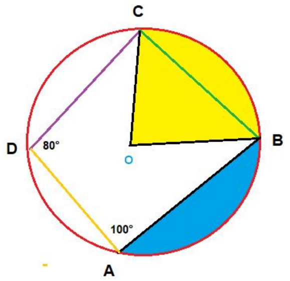
$\Rightarrow$ DF = BC

38. **Read the text carefully and answer the questions:**

There was a circular park in Defence colony at Delhi. For fencing purpose poles A, B, C and D were installed at the circumference of the park.

Ram tied wires From A to B, B to C and C to D, and he managed to measure the $\angle A = 100°$ and $\angle D = 80°$

Point O in the middle of the park is the center of the circle.



(i) ABCD is cyclic quadrilateral.

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.

Here all four vertices A, B, C and D lie on a circle.

(ii) We know that the sum of both pair of opposite angles of a cyclic quadrilateral is 180º.

$\angle C + \angle A = 1800$

$\angle C = 1800 - 1000 = 800$

(iii)We know that

The sum of both pair of opposite angles of a cyclic quadrilateral is 180º.

$\angle B + \angle D = 1800$

$\angle B = 1800 - 800 = 1000$

OR

i. In a cyclic quadrilateral, all the four vertices of the quadrilateral lie on the circumference of the circle.
ii. The four sides of the inscribed quadrilateral are the four chords of the circle.
iii. The sum of a pair of opposite angles is 180° (supplementary). Let $\angle A$, $\angle B$, $\angle C$, and $\angle D$ be the four angles of an inscribed quadrilateral. Then, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.